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Young diagrams, supercharacters of OSp(M/N) and modification rules

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Abstract. The supercharacter of OSp(M/N) associated with an arbitrary Young diagram is defined. The distinction is made between OSp(M/N) standard and non-standard supercharacters. The corresponding modification rule which may be used to express a non-standard supercharacter in terms of standard supercharacters is presented, exemplified and proved. This rule involves the removal of $N/2+1$ boundary strips from the Young diagram. In the case $N=0$ the rule reduces to the well known rule appropriate to O(M). For a non-standard supercharacter corresponding to a typical irreducible representation of OSp(M/N) the modification yields a single typical standard supercharacter. On the other hand, for a non-standard supercharacter corresponding to an atypical irreducible representation the rule yields a linear combination of atypical standard supercharacters.

1. Introduction

The specification of all the inequivalent finite-dimensional irreducible representations of the basic classical Lie superalgebras was completed by Kac (1978) in terms of Kac–Dynkin labels. He also gave an explicit formula for both the character and the supercharacter of each typical irreducible representation whilst pointing out the difficulty of trying to extend this work to cover the case of atypical irreducible representations.

Alternative approaches to this subject (Dondi and Jarvis 1981, Balantekin and Bars 1981a, b, 1982, Bars *et al* 1983, Farmer and Jarvis 1984, Bars 1984, Delduc and Gourdin 1984, 1985, Morel *et al* 1985, Gourdin 1986) have exploited Young diagrams in the case of both U(M/N) and OSp(M/N). This has the merit of leading to certain formulae for the analysis of tensor products and branching rules (Dondi and Jarvis 1981, King 1983, Wybourne 1984) which are very succinct but suffer from the drawback that when they involve atypical irreducible representations they must necessarily be interpreted with considerable care.

Nonetheless the great success of Young diagram methods in accounting, in just this same succinct way, for many properties of irreducible representations of U(M), O(M) and Sp(M) suggests that it is worth trying to generalise from the Lie algebra to the Lie superalgebra context. The key step in the Lie algebra case is to realise that great simplifications occur if M is taken to be arbitrarily large. More precisely, one considers characters depending on a countably infinite set of class variables. The price to be paid for this simplification, however, is that on restriction to a finite set of variables use must be made of modification rules (King 1971, Black *et al* 1983). These arise naturally from the existence of various determinantal expansions of characters of irreducible representations and allow the very efficient evaluation of tensor products, branching rules and symmetrised products (plethysms).

Similar modification rules for OSp(M/N) have recently been proposed without proof (Farmer 1986a, b, King 1986). In this paper a somewhat simpler form of the modification rule is derived. The proof makes use once again of determinantal expansions (El Samra and King 1979). The new rule is shown to be equivalent to the previous more complicated conjectures.

In the next section some notational preliminaries are dealt with and the modification rules appropriate to U(M), O(M) and Sp(M) are written down followed by the modification rule for OSp(M/N) which is the main subject of this paper. It is a moot point whether one chooses to work in terms of characters or supercharacters of OSp(M/N), but it should be stressed that the modification rule obtained here, although expressed in terms of supercharacters, applies equally well to both characters and supercharacters of OSp(M/N). The rationale behind our concentration on supercharacters is the fact that they are associated with the fully OSp(M/N) invariant supertrace operation (Balantekin and Bars 1981a). Nonetheless it is a strength of the Young diagram approach that it is possible to pass from character to supercharacter and vice versa merely by making certain changes of sign. These will be referred to where appropriate in § 3 in which a generating function is used to define the supercharacters of OSp(M/N) associated with Young diagrams, and important results on branching rules and tensor products which follow from this definition are also stated.

In § 4 determinantal expansions of supercharacters are used to recast the required modification rule in a more amenable form, and the rule's validity is finally proved in § 5. Whilst the generating function, the branching rule and some details of the proof differ in various sign factors as between characters and supercharacters, the tensor product rule, the determinantal expansions and the modification rule itself all apply equally well to both characters and supercharacters.

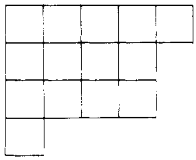
The modification rule applies to all non-standard supercharacters, whether typical or atypical. The distinction between these two cases is made in § 6 where it is shown in particular that each non-standard typical supercharacter is equivalent under modification to a single standard typical supercharacter. This contrasts with the atypical case where after modification a linear combination of standard atypical supercharacters is obtained.

The paper concludes in § 7 with some applications of the modification rule and a warning that much remains to be done in understanding the problems arising from the lack of full reducibility of some representations.

As a final introductory word we would remark that our terminology differs somewhat from that adopted by other authors who make great use of the word 'supertableaux'. Instead we prefer the use of 'Young diagram' to describe the diagram corresponding to a partition label. This allows a distinction to be made, although it is not required in the present paper, between 'diagram' and 'tableaux' where the former consists of boxes or nodes whilst the latter is an array resulting from filling the boxes or replacing the nodes by entries, usually integers, subject to some ordering rules. Thus, partitions label Young diagrams which specify characters or supercharacters, which may be evaluated by enumerating tableaux or supertableaux. It seems worthwhile emphasising this choice of terminology to try and bring into harmony the work of mathematicians and theoretical physicists.

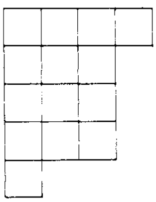
2. Young diagrams and modification rules

Each partition $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)$ with integer parts satisfying $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_p > 0$ serves to specify a Young diagram F^λ consisting of an array of $|\lambda| = \lambda_1 + \lambda_2 + \ldots + \lambda_p$ boxes in p left-adjusted rows of lengths $\lambda_1, \lambda_2, \ldots, \lambda_p$. Typically for $\lambda = (54^21)$.

$F^\lambda =$  (2.1)

The Frobenius rank r of λ is the number of boxes on the main diagonal of F^λ so that in the above example $r = 3$. The conjugate partition λ' is defined to be that partition λ' which specifies the Young diagram $F^{\lambda'}$ obtained from F^λ by interchanging rows and columns.

In the above example

$F^{\lambda'} =$  (2.2)

so that $\lambda' = (43^31)$.

Partitions have been widely used in the specification of irreducible representations of all the classical Lie groups U(M), O(M) and Sp(M). The characters of the covariant tensor irreducible representations of these groups are conveniently denoted by (Littlewood 1950, Black *et al* 1983) $\{\lambda\}$, $[\lambda]$ and $\langle\lambda\rangle$, respectively. These characters and the corresponding Young diagram F^λ are said to be U(M), O(M) and Sp(M) standard if and only if the number of parts, $p = \lambda'_1$, of the partition λ is less than or equal to M, $[M/2]$ and $M/2$, respectively. As noted in the introduction, although it is possible to find formal expressions for Kronecker products, branching rules, etc, when M is arbitrarily large, on restriction to a particular finite value of M some characters will be non-standard and it is necessary to apply modification rules (King 1971, Black *et al* 1983, Koike and Terada 1985) relating them to standard characters. These rules take the form:

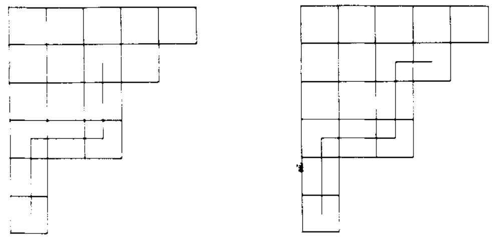
$$\mathrm{U}(M) \qquad \{\lambda\} = 0 \qquad \text{if } h = \lambda'_1 - M - 1 \geqslant 0 \tag{2.3}$$

$$\mathrm{O}(M) \qquad [\lambda] = (-1)^{c+1}\varepsilon[\lambda - h] \qquad \text{if } h = 2\lambda'_1 - M > 0 \tag{2.4}$$

$$\mathrm{Sp}(M) \qquad \langle\lambda\rangle = (-1)^{c}\langle\lambda - h\rangle \qquad \text{if } h = 2\lambda'_1 - M - 2 \geqslant 0 \tag{2.5}$$

where ε is the character of O(M) given by the determinant of each group element and where $\lambda - h$ specifies a diagram $F^{\lambda-h}$ obtained from F^λ by the removal of a continuous boundary strip of boxes of length h starting at the foot of the first column of F^λ and extending over c columns. The boundary strip of length h is said to be removable if $h > 0$ for O(M) and $h \geqslant 0$ for Sp(M) and if the resulting diagram $F^{\lambda-h}$ is regular in the sense that it coincides with a Young diagram F^μ for some partition μ. In such a case $[\lambda - h]$ and $\langle\lambda - h\rangle$ are to be interpreted as $[\mu]$ and $\langle\mu\rangle$, respectively. The boundary strip is said to be not removable if $F^{\lambda-h}$ is irregular in that no partition μ exists for which $F^{\lambda-h} = F^\mu$. In this case $[\lambda - h]$ and $\langle\lambda - h\rangle$ are to be interpreted as being identically zero.

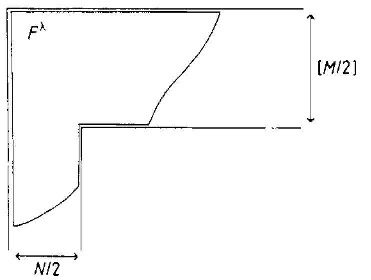
In the case $\lambda = (5\,4\,3^2\,1^2)$, for example, strips of lengths $h = 7$ and 8 are not removable and removable, respectively, as can be seen from the following diagrams



In the latter case $F^{\lambda-h} = F^{\lambda-8} = F^{\mu}$ with $\mu = (5\,2^2)$.

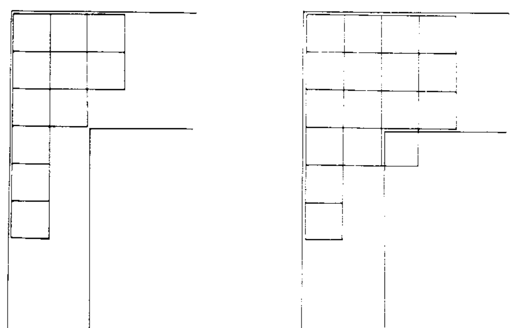
It should be noted that the rules themselves imply the standardness conditions referred to above. If necessary, they are to be applied more than once until $h < 0$ for U(M), $h \leqslant 0$ for O(M) and $h < 0$ for Sp(M) or until the character in question is shown to be zero. In this way each non-standard character is either zero or equivalent to ± 1 or $\pm \varepsilon$ times a standard character. It should be noted that through these modification rules each non-vanishing non-standard character associated with F^{λ} is expressed in terms of a single standard character associated with F^{μ} and correspondingly with a single irreducible representation.

In the case of OSp(M/N) a similar notation and terminology may be employed in that supercharacters associated with covariant tensor representations of OSp(M/N) may be denoted by $[\lambda]$. Such a supercharacter and the corresponding Young diagram F^{λ} are said to be OSp(M/N) standard if and only if $\lambda'_{N/2+1} \leqslant [M/2]$ or equivalently if and only if F^{λ} fits inside a hook-shaped region of arm width $[M/2]$ and leg width $N/2$ as shown:



(2.6)

Thus in the case of OSp(7/4) $[3^2 21^3]$ is standard but $[4^3 31^2]$ is non-standard:



To justify this claim it is of course necessary to provide the modification rule which relates non-standard supercharacters of OSp(M/N) to standard supercharacters. This rule in the form which most closely resembles (2.4) is

$$\mathrm{OSp}(M/N) \qquad [\lambda] = \sum_{k=1}^{l} \sum_{(j)} (-1)^{c_{j_1}+c_{j_2}+\ldots+c_{j_k}+1} \varepsilon^k [\lambda - h_{j_1} - h_{j_2} - \ldots - h_{j_k}] \tag{2.7}$$

with

$$h_j = 2(\lambda_j' - j + 1) - M + N > 0 \tag{2.8}$$

and

$$l = N/2 + 1. \tag{2.9}$$

ε is the supercharacter of OSp(M/N) given by the superdeterminant of each supergroup element, (j) signifies any sequence of integers $(j_1, j_2, \ldots, j_k)$ such that $l \geqslant j_1 > j_2 > \ldots > j_k \geqslant 1$ and the notation is such that $\lambda - h_j$ specifies a diagram $F^{\lambda - h_j}$ obtained from F^λ by the removal of a continuous boundary strip of boxes of length h_j starting at the foot of the jth column and extending over c_j columns to end in the $(j + c_j - 1)$th column. The order in which the strips are removed is immaterial but in order to avoid stating additional rules for overlapping strips it is convenient to start at the rightmost column specified by j_1 and to follow the sequence (j).

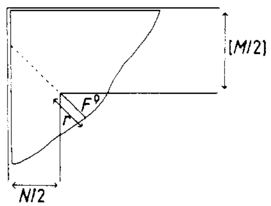
Unlike the O(M) case, for which ε may be identified with the character $[1^M]$ associated with F^{1^M}, in the case of OSp(M/N), ε may not be identified with any supercharacter associated with a Young diagram F^μ.

It is to be noted that the modification rule (2.7) for OSp(M/N) reduces as one would wish to the rule (2.4) for O(M) in the case $N = 0$.

As for O(M) and Sp(M) the modification rule (2.7) for OSp(M/N) may be iterated, leading to the formula:

$$[\lambda] = \sum_{k=r}^{\lambda_1} \sum_{(j)} \binom{k-1}{r-1} (-1)^{c_{j_1}+c_{j_2}+\ldots+c_{j_k}+r} \varepsilon^k [\lambda - h_{j_1} - h_{j_2} - \ldots - h_{j_k}] \tag{2.10}$$

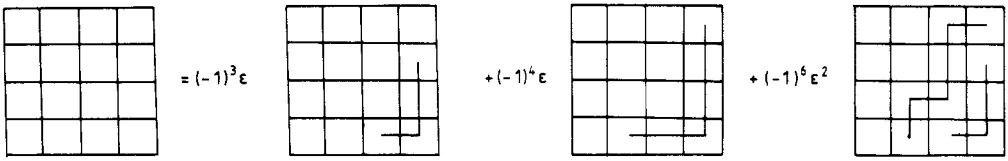
where r is the Frobenius rank of the partition ρ specifying that part F^ρ of F^λ which is OSp(M/N) non-standard, i.e. the non-standard rank of $[\lambda]$:



(2.11)

Note that now the sum extends in general over all columns of F^λ in that (j) signifies any sequence of integers $(j_1, j_2, \ldots, j_k)$ such that $\lambda_1 \geqslant j_1 > j_2 > \ldots > j_k \geqslant 1$. Furthermore each term involves the removal of at least r boundary strips. It is not difficult to see that each removal decreases the Frobenius rank of the non-standard part of the diagram by one. Thus on the right-hand side of (2.10) all the resulting supercharacters are OSp(M/N) standard and no further modification is required or indeed allowed.

As an illustration of the application of (2.7) consider the supercharacter $[4^4]$ of OSp(4/4). In this case $l = N/2 + 1 = 3$ whilst $h_1 = 8$, $h_2 = 6$ and $h_3 = 4$. The strip of length 8 is not removable so that (2.7) yields, in terms of Young diagrams



Hence

$$[4^4] = -\varepsilon[43^22] + \varepsilon[3^31] + \varepsilon^2[2^21^2].$$

The first two terms are still not OSp(4/4) standard. Applying (2.7) again we find

$$[4^4] = -\varepsilon^2[42^3] + \varepsilon^2[32^21] - \varepsilon^2[2^21^2] + 2\varepsilon^3[1^4].$$

This result can be found directly from (2.10) by noting that in this case $r = 2$ and $h_4 = 2$ giving a third removable strip.

3. Supercharacters of OSp(M/N)

The characters of irreducible representations of U(M), O(M) and Sp(M) denoted by $\{\lambda\}$, $[\lambda]$ and $\langle\lambda\rangle$, respectively, are denoted rather more precisely by $\{\lambda; \boldsymbol{x}\}$, $[\lambda; \boldsymbol{x}]$ and $\langle\lambda; \boldsymbol{x}\rangle$, indicating that they are functions of $\boldsymbol{x} = (x_1, x_2, \ldots, x_M)$ where x_i for $i = 1, 2, \ldots, M$ are the eigenvalues of the appropriate group elements realised as $M \times M$ matrices. These characters may be written down by making use of the character formula of Weyl appropriate to all the irreducible representations of all semi-simple Lie groups including the classical ones under consideration here (Weyl 1939).

Such a formula covering all the irreducible representations of all the basic classical simple Lie superalgebras does not exist, although the case of all typical irreducible representations is covered by the supercharacter formula of Kac (1978). As a result it is particularly important when using Young diagram methods to define unambiguously the supercharacter associated with each Young diagram.

The approach adopted here is to make use of generating functions for such supercharacters. These are a natural extension of the following generating functions (Weyl 1939, Littlewood 1950) appropriate to the classical groups which may all be derived from Weyl's character formula:

$$\text{U}(M) \qquad \prod_{i,a} (1 - x_i s_a)^{-1} = \sum_\lambda \{\lambda; \boldsymbol{x}\}\{\lambda; \boldsymbol{s}\} \tag{3.1}$$

$$\text{O}(M) \qquad \prod_{i,a} (1 - x_i s_a)^{-1} \prod_{a \leqslant b} (1 - s_a s_b) = \sum_\lambda [\lambda; \boldsymbol{x}]\{\lambda; \boldsymbol{s}\} \tag{3.2}$$

$$\text{Sp}(M) \qquad \prod_{i,a} (1 - x_i s_a)^{-1} \prod_{a < b} (1 - s_a s_b) = \sum_\lambda \langle\lambda; \boldsymbol{x}\rangle\{\lambda; \boldsymbol{s}\}. \tag{3.3}$$

The last pair of formulae involve the infinite S-function series evaluated by Littlewood (1950, p 238):

$$A(\boldsymbol{x}) = \prod_{i<j} (1 - x_i x_j) = \sum_\alpha (-1)^{|\alpha|/2}\{\alpha; \boldsymbol{x}\} \tag{3.4}$$

$$C(\boldsymbol{x}) = \prod_{i \leqslant j} (1 - x_i x_j) = \sum_\gamma (-1)^{|\gamma|/2}\{\gamma; \boldsymbol{x}\} \tag{3.5}$$

whose inverses are

$$B(x)=\prod_{i<j}(1-x_ix_j)^{-1}=\sum_{\beta}\{\beta;x\} \tag{3.6}$$

$$D(x)=\prod_{i\leq j}(1-x_ix_j)^{-1}=\sum_{\delta}\{\delta;x\}. \tag{3.7}$$

It follows immediately that the characters of O(M) and Sp(M) are related to characters of U(M) by the formulae

$$[\lambda;x]=\{\lambda/C;x\} \tag{3.8}$$

$$\{\lambda;x\}=[\lambda/D;x] \tag{3.9}$$

$$\langle\lambda;x\rangle=\{\lambda/A;x\} \tag{3.10}$$

$$\{\lambda;x\}=\langle\lambda/B;x\rangle. \tag{3.11}$$

The notation for infinite S-function series and operations involving them has been explained elsewhere (King 1975, Black *et al* 1983).

The generating function for the supercharacters of covariant tensor irreducible representations of U(M/N) takes the form (Remmel 1984)

$$\text{U}(M/N)\qquad \prod_{i,a}(1-x_is_a)^{-1}\prod_{j,b}(1-y_jt_b)^{-1}\prod_{i,b}(1-x_it_b)\prod_{j,a}(1-y_js_a)=\sum_{\lambda}\{\lambda;x/y\}\{\lambda;s/t\} \tag{3.12}$$

or more simply

$$\text{U}(M/N)\qquad \prod_{i,a}(1-x_is_a)^{-1}\prod_{j,a}(1-y_js_a)=\sum_{\lambda}\{\lambda;x/y\}\{\lambda;s\} \tag{3.13}$$

where $\{\lambda;x/y\}$ denotes the supercharacter of U(M/N) corresponding to the Young diagram F^λ.

The analogous generating function for supercharacters of OSp(M/N), which do not necessarily correspond to irreducible representations, takes the form

$$\begin{aligned}\text{OSp}(M/N)\qquad &\prod_{i,a}(1-x_is_a)^{-1}\prod_{j,c}(1-y_jt_c)^{-1}\prod_{i,c}(1-x_it_c)\prod_{j,a}(1-y_js_a)\\ &\quad\times\prod_{a\leq b}(1-s_as_b)\prod_{c<d}(1-t_ct_d)\prod_{a,c}(1-s_at_c)^{-1}\\ &=\sum_{\lambda}[\lambda;x/y]\{\lambda;s/t\}\end{aligned} \tag{3.14}$$

or more simply

$$\text{OSp}(M/N)\qquad \prod_{i,a}(1-x_is_a)^{-1}\prod_{j,a}(1-y_js_a)\prod_{a\leq b}(1-s_as_b)=\sum_{\lambda}[\lambda;x/y]\{\lambda;s\}. \tag{3.15}$$

In these formulae (3.12)-(3.15) $x=(x_1,x_2,\ldots x_M)$ and $y=(y_1,y_2,\ldots,y_N)$ are such that, for $i=1,2,\ldots,M$ and $j=1,2,\ldots,N$, x_i and y_j are eigenvalues of the $M\times M$ and $N\times N$ matrices, respectively, of the Lie supergroups U(M/N) and OSp(M/N) restricted to U(M)×U(N) and O(M)×Sp(N).

Having defined the required supercharacter of OSp(M/N) by means of (3.15) it follows from (3.5) by the same arguments as in the Lie algebra case that

$$[\lambda;x/y]=\{\lambda/C;x/y\} \tag{3.16}$$

and also

$$\{\lambda; x/y\} = [\lambda/D; x/y]. \tag{3.17}$$

The application of (3.1) and the related identity

$$\prod_{i,a} (1 - x_i s_a) = \sum_{\lambda} (-1)^{|\lambda|}\{\lambda; x\}\{\lambda'; s\} \tag{3.1'}$$

to (3.13) immediately gives an expansion of the supercharacter $\{\lambda; x/y\}$ of $U(M/N)$ in terms of characters of $U(M)$ and $U(N)$

$$\{\lambda; x/y\} = \sum_{\mu} (-1)^{|\mu|}\{\lambda/\mu; x\}\{\mu'; y\}. \tag{3.18}$$

Similarly (3.15) may be used to derive the expansion of $[\lambda; x/y]$ in terms of characters of $O(M)$ and $Sp(N)$

$$[\lambda; x/y] = \sum_{\mu} (-1)^{|\mu|}[\lambda/\mu D; x]\langle\mu'; y\rangle. \tag{3.19}$$

It should be noted that these two results (3.18) and (3.19), which follow from the definition of supercharacters based on the generating functions (3.13) and (3.15) respectively, could themselves serve to define the supercharacters. It is also worth pointing out that the relationship between characters and supercharacters of Lie superalgebras (Kac 1978) is such that for both $U(M/N)$ and $OSp(M/N)$ it is only necessary to change every indeterminate y_j to $-y_j$ in (3.13), (3.15), (3.18) and (3.19) to pass from supercharacters to characters. In particular, this means that the characters of $U(M/N)$ and $OSp(M/N)$ associated with the supercharacters $\{\lambda; x/y\}$ and $[\lambda; x/y]$, respectively, are defined by (3.18) and (3.19) simply through the omission of the sign factors $(-1)^{|\mu|}$ in agreement with branching rules given earlier (King 1983, Wybourne 1984). In what follows we have chosen to consistently work in terms of supercharacters but as stressed in the introduction the resulting modification rule (2.7) is true for both characters and supercharacters.

If we now take x and y to be infinite sets of variables $x = (x_1, x_2, \ldots)$ and $y = (y_1, y_2, \ldots)$ then the following result holds (Cummins 1986):

$$[\lambda; x/y] \times [\mu; x/y] = \sum_{\nu} [(\lambda/\nu)(\mu/\nu); x/y]. \tag{3.20}$$

This has exactly the same form as that for products of orthogonal and symplectic characters of an infinite set of variables:

$$[\lambda; x] \times [\mu; x] = \sum_{\nu} [(\lambda/\nu)(\mu/\nu); x] \tag{3.21}$$

$$\langle\lambda; x\rangle \times \langle\mu; x\rangle = \sum_{\nu} \langle(\lambda/\nu)(\mu/\nu); x\rangle. \tag{3.22}$$

The difference between these expressions lies in their restriction to a finite number of variables. In the case of (3.21) and (3.22) non-standard terms on the right-hand side will have to be modified using (2.4) and (2.5). In exactly the same manner non-standard terms in (3.20) will have to be modified by the use of (2.7) which is still to be derived.

4. Determinantal expansions

The key to proving the modification rule for O(M) is the exploitation of the determinantal expansion (King 1971):

$$\mathrm{O}(M) \qquad [\lambda]=|[1^{\lambda'_j-j+i}]+(1-\delta_{i1})[1^{\lambda'_j-j-i+2}]| \tag{4.1}$$

where

$$[1^t]=0 \qquad \text{for } t<0 \tag{4.2}$$

together with the fundamental modification rule

$$[1^{M-t}]=\varepsilon[1^t] \qquad \text{for all } t. \tag{4.3}$$

This implies also that

$$[1^t]=0 \qquad \text{for } t>M. \tag{4.4}$$

Equation (4.3) may be easily understood in terms of contracting the t indices of an antisymmetric tensor representation of O(M) corresponding to $[1^t]$ with the Mth rank totally antisymmetric tensor $\varepsilon_{i_1 i_2 \ldots i_M}$ used in defining the determinant of a group element (for a more formal proof see Koike and Terada (1985)). The lack of such a tensor for OSp(M/N), with $N>0$, and the consequent lack of a fundamental modification rule such as (4.3) accounts for the somewhat indirect proof of the OSp(M/N) modification rule (2.7) presented here.

The starting point is the analogue of (4.1):

$$\mathrm{OSp}(M/N) \qquad [\lambda]=|[1^{\lambda'_j-j+i}]+(1-\delta_{i1})[1^{\lambda'_j-j-i+2}]|. \tag{4.5}$$

In the case where the number of variables is infinite this follows from (4.1) because of the identical products (3.20) and (3.21) (Cummins 1986). Restricting to a finite set of variables preserves the form of (4.5), and in particular this identity is valid for non-standard $[\lambda; x/y]$.

In order to make use of (4.5) note that the required modification rule (2.7) can first be recast in the form

$$\sum_{k=0}^{l}\sum_{(j)}(-1)^{c_{j_1}+c_{j_2}+\ldots+c_{j_k}}\varepsilon^k[\lambda-h_{j_1}-h_{j_2}-\ldots-h_{j_k}]=0. \tag{4.6}$$

Quite generally, however, the removal of an individual continuous boundary strip of length h_j from F^λ amounts to replacing the parameter λ'_j appearing in the jth column of the determinantal expansion of $[\lambda]$ by λ'_j-h_j and then reordering the columns of the determinant (King 1971). This reordering in the case of O(M) is based on the fact that $[\lambda]$ depends on column lengths through the factor λ'_j-j in (4.1) and the antisymmetry of the determinant under transpositions of its columns. The same is true in the case of OSp(M/N) as can be seen from (4.5). Hence, making use of (4.5) for each of the supercharacters appearing in (4.6), the modification rule can be recast once again, this time as

$$|[1^{\lambda'_j-j+i}]+(1-\delta_{i1})[1^{\lambda'_j-j-i+2}]-\varepsilon\theta_{jl}([1^{M-N-\lambda'_j+j-i}]+(1-\delta_{i1})[1^{M-N-\lambda'_j+j+i-2}])|=0 \tag{4.7}$$

where as usual $l=N/2+1$ and

$$\theta_{jl}=\begin{cases}1 & \text{if } 1\leqslant j\leqslant l\\ 0 & \text{if } j>l.\end{cases} \tag{4.8}$$

This is an identity of the form

$$\det \mathscr{D} = 0 \tag{4.9}$$

where the ijth entry of the matrix $\mathscr{D}$ has been displayed and $1 \leqslant i, j \leqslant m$ where $m \geqslant \lambda_1$, the number of columns of F^λ. In fact, in what follows it is also necessary to ensure that $m \geqslant N+1$ but this is easily done just by remembering that any columns of F^λ have length $\lambda'_j = 0$ for $j > \lambda_1$. Notice also that the replacement of λ'_j by

$$\lambda'_j - h_j = M - N - \lambda'_j + 2j - 2 \tag{4.10}$$

turns out to be exactly equivalent to modifying the entries of (4.5) as if they were characters of $O(M-N)$ to produce via (4.3) the modified terms appearing in (4.7).

Now that the modification rule has been given in terms of the matrix $\mathscr{D}$ its validity will be established by first branching from $OSp(M/N)$ to $O(M) \times Sp(N)$ and then finding an explicitly non-singular matrix $\mathscr{M}$ such that $\mathscr{M}\mathscr{D}$ is manifestly singular. The branching from $OSp(M/N)$ to $O(M) \times Sp(N)$ is accomplished by noting the special case of (3.19):

$$[1^p] = \sum_{t=-\infty}^{\infty} (-1)^{p-t}[1^t] \times \langle p-t \rangle \tag{4.11}$$

and the restricting of the superdeterminant

$$\varepsilon = \varepsilon \times \langle 0 \rangle. \tag{4.12}$$

In (4.11) the sum has been extended from $t = -\infty$ to $t = +\infty$ through the use of (4.2) and (4.4).

5. Derivation of the modification rule for OSp(M/N)

The final ingredients are some results on characters of $Sp(N)$ which will be required later. This time the key is the determinantal expansion due to Weyl (1939, p 219)

$$Sp(N) \qquad \langle \lambda \rangle = |\{\lambda_i - i + j\} + (1 - \delta_{ij})\{\lambda_i - i - j + 2\}| \tag{5.1}$$

where $\{\mu\}$ signifies the character of an irreducible representation of $U(N)$ restricted to $Sp(N)$. Expansion of this determinant with respect to the elements in the first row leads to the following expression (El Samra and King 1979) appropriate to characters specified by Young diagrams consisting of a single hook:

$$Sp(N) \qquad \langle p, 1^{q-1} \rangle = \sum_{i=1}^{q} (-1)^{i+1}(\{p-1+i\} + (1-\delta_{i1})\{p+1-i\})\{1^{q-i}\} \tag{5.2}$$

where on the right-hand side the $U(N)$ characters are given in terms of $Sp(N)$ characters by

$$\{s\} = \langle s \rangle \tag{5.3}$$

$$\{1^s\} = \sum_{t=0}^{\infty} \langle 1^{s-2t} \rangle \tag{5.4}$$

where of course

$$\langle 1^s \rangle = 0 \qquad \text{for} \qquad s < 0. \tag{5.5}$$

In what follows non-standard expressions of the form (5.2) arise and it is important to note that $\langle p, 1^{q-1}\rangle$ is Sp(N) standard if and only if either $p=0$ and $q=1$ or $p\geqslant 1$ and $1\leqslant q\leqslant N/2$. If $p\geqslant 1$ and $q\geqslant N/2+1$ the following modifications apply as can be seen from (2.5) with M replaced by N:

$$\langle p, 1^{q-1}\rangle = -\langle p, 1^{N+1-q}\rangle \qquad \text{for } N/2\leqslant q-1\leqslant N \tag{5.6}$$

and

$$\langle p, 1^{q-1}\rangle = \begin{cases}(-1)^p\langle 0\rangle & \text{for } q-1\geqslant N+1 \text{ and } p=q-1-N\\ 0 & \text{for } q-1\geqslant N+1 \text{ and } p\neq q-1-N.\end{cases} \tag{5.7}$$

Finally for $p\leqslant 0$ and $q-1\geqslant 0$

$$\langle p, 1^{q-1}\rangle = \begin{cases}(-1)^p\langle 0\rangle & \text{for } p=-(q-1)\\ 0 & \text{for } p\neq -(q-1)\end{cases} \tag{5.8}$$

as can be derived from (5.2) or indeed from a consideration of (5.1).

Now to the required proof. First define a column matrix $\mathscr{C}^p$ whose ith element for $1\leqslant i\leqslant m$ is given in terms of OSp(M/N) supercharacters by

$$\mathscr{C}^p_i = [1^{p-1+i}] + (1-\delta_{i1})[1^{p+1-i}] \tag{5.9}$$

where p is any integer. The restriction to O(M)$\times$Sp(N) is accomplished using (4.11) and yields

$$\mathscr{C}^p_i = \sum_{t=-\infty}^{\infty} (-1)^{p-1+i-t}[1^t]\times(\langle p-1+i-t\rangle + (1-\delta_{i1})\langle p+1-i-t\rangle) \tag{5.10}$$

so that from (5.3)

$$\mathscr{C}^p_i = \sum_{t=-\infty}^{\infty} (-1)^{p-t}[1^t]\times(-1)^{i+1}(\{p-1+i-t\} + (1-\delta_{i1})\{p+1-i-t\}). \tag{5.11}$$

Next define a row matrix $\mathscr{R}_q$ where q is any integer and whose ith element for $1\leqslant i\leqslant m$ is given in terms of O(M)$\times$Sp(N) characters by

$$\mathscr{R}^i_q = \sum_{s=0}^{\infty} [0]\times\langle 1^{q-i-2s}\rangle = [0]\times\{1^{q-i}\} \tag{5.12}$$

where use has been made of (5.4). Notice that

$$\mathscr{R}^i_q = \begin{cases}1 & \text{for } i=q\\ 0 & \text{for } i>q\end{cases} \tag{5.13}$$

then

$$\begin{aligned}\mathscr{R}_q\mathscr{C}^p &= \sum_i \mathscr{R}^i_q\mathscr{C}^p_i\\ &= \sum_{t=-\infty}^{\infty} (-1)^{p-t}[1^t]\times\langle p-t, 1^{q-1}\rangle\end{aligned} \tag{5.14}$$

where use has been made of (5.2).

To make contact with the matrix $\mathscr{D}$ it is to be noted that

$$\mathscr{D}_{ij} = \mathscr{C}_i^{\lambda'_j-j+1} - \varepsilon\theta_{jl}\mathscr{C}_i^{M-N-\lambda'_j+j-1} \tag{5.15}$$

whilst the matrix $\mathscr{M}$ used to show that $\mathscr{D}$ is singular is defined by

$$\mathscr{M}_{ki} = \mathscr{R}^i_{m-k+1} + \theta_{k,m-l+1}\mathscr{R}^i_{N-m+k+1}. \tag{5.16}$$

In these expressions i, j, k take on the values $1, 2, \ldots, m$ whilst $l = N/2+1$ and $m \geqslant \lambda_1$ and $m \geqslant N+1$. The matrix $\mathcal{M}$ then takes the form

$$\mathcal{M} = \left[\begin{array}{ccccccc} [0]\times\{1^{M-1}\} & [0]\times\{1^{M-2}\} & & \cdots & & \cdots & [0]\times\{0\} \\ [0]\times\{1^{M-2}\} & [0]\times\{1^{M-3}\} & & \cdots & & [0]\times\{0\} & 0 \\ \vdots & \vdots & & & & & \vdots \\ \vdots & [0]\times\{0\} & 0 & \cdots & & & 0 \\ [0]\times\{0\} & 0 & 0 & \cdots & & & 0 \end{array}\right] \Big\updownarrow m$$

$$+ \left[\begin{array}{cccccccc} 0 & 0 & 0 & \cdots & & & & 0 \\ \vdots & \vdots & \vdots & & & & & \vdots \\ 0 & 0 & 0 & \cdots & & & & 0 \\ [0]\times\{0\} & 0 & 0 & \cdots & & & & 0 \\ [0]\times\{1\} & [0]\times\{0\} & 0 & \cdots & & & & 0 \\ \vdots & \vdots & & & & & & \\ [0]\times\{1^{N/2}\} & [0]\times\{1^{N/2-1}\} & \cdots & [0]\times\{0\} & 0 & & \cdots & 0 \\ 0 & 0 & & 0 & 0 & & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & & & \vdots \\ 0 & 0 & & 0 & 0 & & \cdots & 0 \end{array}\right] \begin{array}{l} \Big\updownarrow m-N-1 \\ \\ \Big\updownarrow N/2+1 \\ \\ \Big\updownarrow N/2 \end{array} \qquad (5.17)$$

$$\underbrace{\qquad\qquad\qquad}_{N/2+1} \qquad \underbrace{\qquad\qquad}_{m-N/2+1}$$

Since m has been chosen so that not only is $m \geqslant \lambda_1$ but also $m \geqslant N+1$ it is then easy to see that $\mathcal{M}$ is non-singular.

The particular values of k and j of interest in the product matrix

$$\sum_{i=1}^{m} \mathcal{M}_{ki} \mathcal{D}_{ij} = \mathcal{P}_{kj} \qquad (5.18)$$

are those for which

$$1 \leqslant k \leqslant m - l + 1 = m - N/2 \qquad \text{and} \qquad 1 \leqslant j \leqslant l = N/2+1. \qquad (5.19)$$

These values of k can be conveniently subdivided into two sets according as

$$1 \leqslant k \leqslant m - N - 1 \qquad (5.20)$$

or

$$m - N \leqslant k \leqslant m - N/2. \qquad (5.21)$$

For the first of these, (5.20), the structure of $\mathcal{M}$ simplifies so that by (5.13) and (5.16), or equivalently from (5.18)

$$\mathcal{M}_{ki} = \mathcal{R}^{i}_{m-k+1}. \qquad (5.22)$$

With this constraint (5.14), (5.15) and (5.22) imply that

$$\mathcal{P}_{kj} = \sum_{t=-\infty}^{\infty} ((-1)^{p-t}[1^t] \times \langle p-t, 1^{m-k}\rangle - (-1)^{M-N-p-t}[1^{M-t}]$$
$$\times \langle M-N-p-t, 1^{m-k}\rangle) \qquad (5.23)$$

with $p=\lambda_j'-j+1$, and use has been made of both (4.12) and (4.3). For certain values of t it is clear that $p-t\leqslant 0$ and for others $M-N-p-t\leqslant 0$ and it is then necessary to modify using (5.8). Moreover in all the remaining terms $m-k\geqslant N+1$ so that use must then be made of (5.6). The surviving terms are very simple and give

$$\mathscr{P}_{kj}=[1^{m+p-k}]\times\langle 0\rangle+[1^{N+p-m+k}]\times\langle 0\rangle-[1^{N+p-m+k}]\times\langle 0\rangle-[1^{m+p-k}]\times\langle 0\rangle \tag{5.24}$$

with the various contributions arising in turn from $t=m+p-k$, $t=N+p-m+k$, $M-t=N+p-m+k$ and $M-t=m+p-k$. It follows that

$$\mathscr{P}_{kj}=0 \qquad \text{for } 1\leqslant k\leqslant m-N-1 \text{ and } 1\leqslant j\leqslant l=N/2+1. \tag{5.25}$$

Turning to the second set of values of k as defined by (5.21) it follows from (5.14)-(5.16) that

$$\begin{aligned}\mathscr{P}_{kj}=\sum_{t=-\infty}^{\infty}&((-1)^{p-t}[1^t]\times\langle p-t,1^{m-k}\rangle-(-1)^{M-N-p-t}[1^{M-t}]\times\langle M-N-p-t,1^{m-k}\rangle\\&+(-1)^{p-t}[1^t]\times\langle p-t,1^{N-m+k}\rangle-(-1)^{M-N-p-t}[1^{M-t}]\\&\times\langle M-N-p-t,1^{N-m+k}\rangle)\end{aligned} \tag{5.26}$$

where $p=\lambda_j'-j+1$. Once more for certain values of t it is clear that $p-t\leqslant 0$ and for others $M-N-p-t\leqslant 0$ and recourse must be made to (5.8). Now, however, for the remaining terms (5.21) implies $N/2\leqslant m-k\leqslant N$ so that (5.6) must be used giving

$$\langle p-t,1^{m-k}\rangle=-\langle p-t,1^{N-m+k}\rangle \tag{5.27}$$

and

$$\langle M-N-p-t,1^{m-k}\rangle=-\langle M-N-p-t,1^{N-m+k}\rangle. \tag{5.28}$$

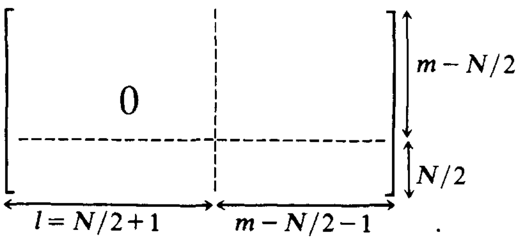
Thus the corresponding contributions from the first and third terms of (5.26) and from the second and fourth terms mutually cancel. Thus

$$\mathscr{P}_{kj}=0 \qquad \text{for } m-N\leqslant k\leqslant m-N/2 \text{ and } 1\leqslant j\leqslant l=N/2+1. \tag{5.29}$$

Combining (5.25) and (5.29) gives

$$\mathscr{P}_{kj}=0 \qquad \text{for } 1\leqslant k\leqslant m-N/2 \text{ and } 1\leqslant j\leqslant N/2+1 \tag{5.30}$$

so that $\mathscr{M}\mathscr{D}=\mathscr{P}$ has the structure

$\mathscr{P}=$ 

(5.31)

It follows that

$$\det\mathscr{P}=\det\mathscr{M}\mathscr{D}=0. \tag{5.32}$$

However $\mathscr{M}$ is non-singular so that, at last,

$$\det\mathscr{D}=0 \tag{5.33}$$

and the modification rule (4.7) and hence (2.7) is proved.

It is perhaps worth pointing out that the proof depends on l (which we have chosen to be $N/2+1$) being greater than $N/2$, but the argument can be carried through for all l such that $N/2<l\leqslant m$. In particular (2.7) is valid for $l=\lambda_1$ and it was in this form that the modification rule was conjectured elsewhere (King 1986) to be true.

The alternative conjecture of Farmer (1986b) can also be verified by noting that just as the iteration of (2.7) gives (2.10) so the iteration of Farmer's more complicated looking formula also leads inexorably to (2.10).

The great merit of (2.7) is that it is the direct generalisation to $\mathrm{OSp}(M/N)$ of the result (2.4) appropriate to $\mathrm{O}(M)$. Indeed as pointed out earlier (2.4) can be recovered from (2.7) by simply setting $N=0$. Furthermore, taking $l=N/2+1$ leads to the smallest possible number of supercharacters in modifying a non-standard supercharacter, namely $2^{N/2+1}-1$ if all the continuous boundary strips are removable. On the other hand, further modifications may be required, leading ultimately to (2.10).

6. Typical and atypical supercharacters

As pointed out by Kac (1978) a distinction must be drawn between typical and atypical irreducible representations. This distinction shows itself in dealing with and indeed in modifying non-standard supercharacters of $\mathrm{OSp}(M/N)$ even though the formula (2.7) is valid in all cases.

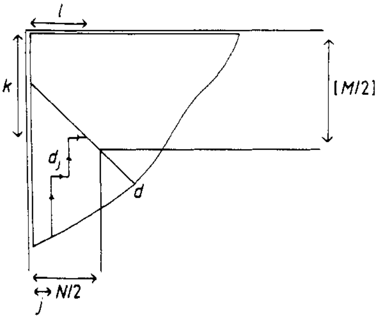
To see the distinction it is necessary to discuss the geometry of the removal procedure. A necessary condition for a strip to be removable is that

$$h_j = 2(\lambda'_j - j + 1) - M + N > 0. \tag{6.1}$$

This quantity h_j can be given a geometrical interpretation. To this end consider the diagonal d passing through the corner box in the non-standard portion F^ρ of F^λ, i.e. the box in row $[M/2]+1$ and column $N/2+1$. Then the distance from the box at the foot of the jth column of F^λ to any box on this diagonal in the kth row and lth column with $k \leqslant \lambda'_j$ and $l \geqslant j$ is given by

$$d_j = \lambda'_j - [M/2] + (N/2 - j) + 1 \tag{6.2}$$

measuring distance upwards and to the right as shown below:



Comparing (6.1) and (6.2)

$$h_j = \begin{cases} 2d_j & \text{if } M \text{ is even} \\ 2d_j - 1 & \text{if } M \text{ is odd.} \end{cases} \tag{6.3}$$

The significance of this is firstly that the strip of length h_j is removable only if $d_j \geqslant 1$ and secondly that the removal process necessarily reduces by one the number of boxes on the diagonal d under consideration. It was this observation which led to the claims made in § 2 regarding (2.10) and the connection between non-standardness and the Frobenius rank r of the non-standard part F^ρ of F^λ shown in (2.11) with the diagonal d indicated.

A further significant geometrical aspect of the removal procedure concerns the regularity or otherwise of $F^{\lambda-h_j}$. This will be regular if and only if the strip extends from the foot of the jth column of F^{λ} to the end of some row, say the ith. Thus if h_j is to be removable there must exist i such that

$$h_j = \lambda'_j + \lambda_i - j - i + 1 \tag{6.4}$$

where the expression on the right-hand side is the familiar hook length h_{ij} associated with the box at the intersection of the ith row and jth column.

Combining (6.1) and (6.4) it follows that

$$(\lambda_i - i + 1) - (\lambda'_j - j + 1) = N - M + 1. \tag{6.5}$$

The significance of this result arises from the distinction between typical and atypical irreducible representations. For standard supercharacters of OSp(M/N) specified by Young diagrams the atypicality conditions of Kac (1978) have been written down (Farmer and Jarvis 1984, Morel *et al* 1984). Their results can be summarised by stating that the supercharacter $[\lambda]$ of OSp(M/N) is typical if and only if none of the following atypicality conditions is satisfied:

$$\lambda'_j + \nu_i + N/2 = i + j - 1 \qquad \text{for } 1 \leq i \leq [M/2] \text{ and } 1 \leq j \leq N/2 \tag{6.6}$$

$$\lambda'_j + N/2 + i + 1 = \nu_i + M + j \qquad \text{for } 1 \leq i \leq [M/2] \text{ and } 1 \leq j \leq N/2 \tag{6.7}$$

with

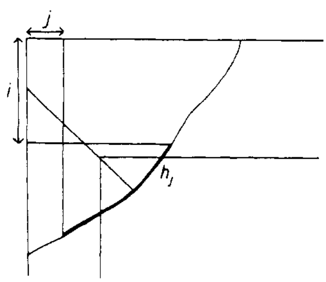
$$\nu_i = \begin{cases} \lambda_i - N/2 & \text{if } \lambda_i \geq N/2 \\ 0 & \text{if } \lambda_i < N/2. \end{cases} \tag{6.8}$$

Extending this terminology to include non-standard supercharacters $[\lambda]$ it is clear that $\lambda_i \geq N/2$ for all $i = 1, 2, \ldots, [M/2]$. Hence ν_i can be replaced in (6.6) and (6.7) by $\lambda_i - N/2$ giving

$$(\lambda_i - i + 1) + (\lambda'_j - j + 1) = 0 \qquad \text{for } 1 \leq i \leq [M/2] \text{ and } 1 \leq j \leq N/2 \tag{6.9}$$

$$(\lambda_i - i + 1) - (\lambda'_j - j + 1) = N - M + 1 \qquad \text{for } 1 \leq i \leq [M/2] \text{ and } 1 \leq j \leq N/2. \tag{6.10}$$

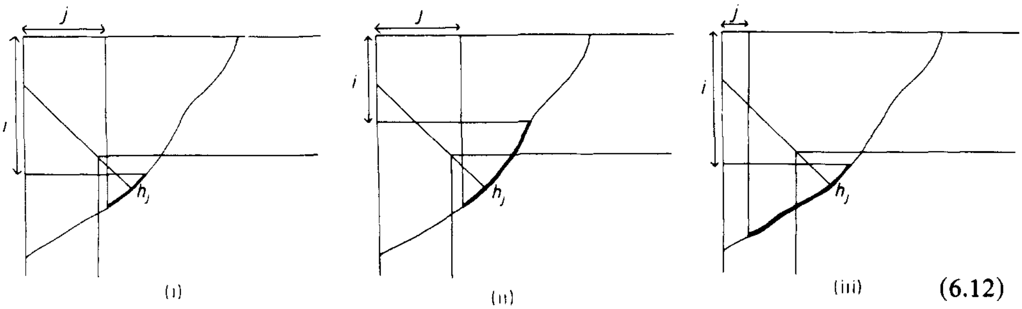
The condition (6.9) can never be satisfied by a non-standard supercharacter since the left-hand side is the hook length h_{ij} of the box in the ith row and jth column and is necessarily positive. Thus a non-standard supercharacter $[\lambda]$ might be said to be typical if and only if none of the atypicality conditions (6.10) is satisfied. For the moment we do this although we are shortly led to a better definition of typicality in the non-standard case. Comparison with (6.5) shows that for a typical non-standard supercharacter none of the removable strips h_j can extend from the jth column with $1 \leq j \leq N/2$ to the ith row with $1 \leq i \leq [M/2]$, i.e. pass from the leg of the diagram F^{λ} to the arm:



(6.11)

Conversely for an atypical non-standard supercharacter at least one such removable strip does do so.

In the case of a typical non-standard supercharacter therefore all the strip removals must be one or other of the three forms shown below:



(6.12)

with either

$$\begin{array}{llll} \text{(i)} & i>[M/2] & \text{and} & j>N/2 \\ \text{(ii)} & 1\leqslant i\leqslant [M/2] & \text{and} & j>N/2 \\ \text{(iii)} & i>[M/2] & \text{and} & 1\leqslant j\leqslant N/2 \end{array} \qquad (6.13)$$

where all strips intersect the diagonal d as explained earlier. However, at the end of the iteration procedure this diagonal must be intersected at least r times where r is the non-standard rank of $[\lambda]$. Thus at least r strips must be removed. However, no more than r may be removed for otherwise a removal of the form (6.11) must occur, but that is not allowed for the typical non-standard supercharacters under discussion. The upshot of this is that either $[\lambda]$ is identically zero or there exist precisely r removable strips and (2.10) yields just a single standard supercharacter on the right-hand side.

Thus non-standard typical supercharacters $[\lambda]$ do indeed modify to give standard supercharacters $[\mu]$ which are themselves typical since we still have $\mu_i \geqslant N/2$ and (6.9) and (6.10) are never satisfied with λ replaced by the new partition μ. Furthermore it is not difficult to see that for each such $[\lambda]$ all the removals are of the form specified by (6.13) (i) and (ii) or all of the form specified by (6.13) (i) and (iii). Moreover in these two cases the final modification can be seen to be equivalent to modifying the portion of F^λ to the right of the leg of width $N/2$ with respect to O(M) and the conjugate of the portion below the arm of width $[M/2]$ with respect to Sp(N) or O($N+1$) according as M is even or odd, respectively. This confirms the result of Farmer (1986a) expressed in this way. It remains to be proved that the standard typical supercharacters $[\lambda]$ do indeed coincide with the supercharacters of typical irreducible representations derived by Kac (1978), although it can be expected with considerable confidence that they do coincide as is implicit in previous publications (Farmer and Jarvis 1984, Morel *et al* 1985).

It should be noted that what we have previously called an atypical non-standard supercharacter $[\lambda]$ possessing non-standard rank r and a total of s removable strips under modification by (2.10) yields zero if $s<r$, a single typical standard supercharacter if $s=r$ and a linear combination of atypical standard supercharacters if $s>r$. This observation leads us to the improvement in definition referred to earlier, namely a supercharacter $[\lambda]$ of OSp(M/N) is said to be

standard if $r\leqslant 0$ and non-standard if $r>0$

and

typical if $r=s$, atypical if $r<s$ and zero if $r>s$

where r is the OSp(M/N) non-standard rank of $[\lambda]$ defined by

$$r = \text{card}\{j: \lambda'_j \geq [M/2] - N/2 + j, j = 1, 2, \ldots\} - N/2 \tag{6.14}$$

and s is the number of removable strips of $[\lambda]$ defined by

$$s = \text{card}\{(i, j); 2(\lambda'_j - j + 1) - M + N = \lambda_i + \lambda'_j - i - j + 1 > 0\}. \tag{6.15}$$

7. Application of the modification rule

In conclusion we illustrate the use of the modification rules in dealing with OSp(M/N) non-standard supercharacters. The first example is provided by the use of the branching rule

$$\text{U}(M/N) \to \text{OSp}(M/N) \qquad \{\lambda\} \to [\lambda/D] \tag{7.1}$$

which follows from (3.18). The supercharacter $\{32^2\}$ of U(3/2) is both standard and typical. However on restricting to OSp(3/2) (7.1) gives

$$\{32^2\} \to [32^2] + [32] + [2^21] + [3] + [21] + [1]. \tag{7.2}$$

The first three supercharacters are non-standard typical supercharacters of OSp(3/2) and the use of (2.7) gives

$$\{32^2\} \to \varepsilon[1^2] + \varepsilon[31] + \varepsilon[21^2] + [3] + [21] + [1]. \tag{7.3}$$

As a second example non-standard supercharacters arise through the application of the product rule (3.20). This is illustrated by

$$[2] \times [2] = [4] + [31] + [2^2] + [2] + [1^2] + [0] \tag{7.4}$$

where $[2^2]$ is non-standard and atypical in OSp(3/2). Carrying out the modification using (2.7) gives

$$[2] \times [2] = [4] + [31] + \varepsilon[21] - \varepsilon[1] + \varepsilon^2[0] + [2] + [1^2] + [0]. \tag{7.5}$$

Clearly the results (7.3) and (7.5) still need careful interpretation. They both involve standard but atypical supercharacters. These are an indication that not fully reducible representations may be present reinforced by the minus sign appearing in (7.5) which makes it manifestly clear that not all supercharacters, even when standard, correspond in a one-to-one way with irreducible representations. Indeed in this example, $[21] - [1]$ is the supercharacter of a single irreducible atypical representation of OSp(3/2) with highest weight $[1] \times \langle 2 \rangle$, expressed in terms of O(3) × Sp(2) characters (Farmer and Jarvis 1984, Morel *et al* 1985).

This illustrates that, although the modification rule we have derived applies to all non-standard supercharacters whether typical or atypical, it cannot by its very nature give global information on the non-fully reducible problem in OSp(M/N) since it only involves functions, supercharacters, evaluated on O(M) × Sp(N). It is hoped, however, that the detailed knowledge that we have obtained concerning the modification rules will be of use in the study of atypical irreducible representations.

Acknowledgments

Thanks are due to Dr R J Farmer for drawing our attention to the fact that with our original terminology an atypical non-standard supercharacter may give, under modification, a typical standard supercharacter. This has been remedied by the adoption of the definitions concluding § 6. One of us (CJC) would like to thank the SERC for financial support under grant no 833101519.

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